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THE CONSTRUCTION OF THE OPTIMAL CONTOUR OF THE LEADING EDGE OF A BODY IN A SUPERSONIC FLOW†

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The problem of the profiling of the contour of the leading edge of a plane body which, on joining the initial and final fixed points, gives minimum drag in a uniform supersonic flow of an ideal (inviscid and non-heat-conducting) gas is considered. According to previous investigations, it is close to a segment of a straight line in that part of the space D of the governing parameters of the problem (the Mach number M_∞, or the dimensionless velocity of the free stream V_∞, the relative thickness τ, and so on) for which there is an attached shock wave in a flow past the required contour. By making use of this fact one can find the "main correction" to the rectilinear generatrix in an explicit form and represent the characteristics of practically optimal leading edges in the form of isolines in the $V_\infty\tau$-plane. The approach naturally leads to an exact result in the case of a rectilinear optimal generatrix (a wedge). It is well known that a wedge in the body of minimum drag for zero reflection coefficient λ of pressure perturbations from the oblique shock wave which occurs in a flow past a wedge. It is shown that the above-mentioned possibility is not unique. In addition to the case when $\lambda(V_\infty, \tau) = 0$ the rectilinear generatrix is also optimal for $\lambda \neq 0$ when the flow beyond the oblique shock wave is sonic.

1. Let x, y be Cartesian coordinates with the x-axis directed along the velocity vector $\mathbf{V}_\infty$ of a uniform supersonic flow (Fig. 1a, where the double line is the shock wave and the thin lines are the c^+- and c^--characteristics). Let the origin of coordinates coincide with the initial point i of the required generatrix if, and, by virtue of this: $x_i = y_i = 0, x_f = X$ and $y_f = \tau X$ for specified X and τ. Henceforth, the subscript ∞ is assigned to the parameters of the free stream and the letters $i, f, \ldots$ to quantities at the corresponding points. For the velocity, density ρ and pressure p scales, we take V_∞°, ρ_∞° and $\rho_\infty^\circ V_\infty^{\circ 2}$, where $V = |\mathbf{V}|$, and the index $^\circ$ is attached to dimensional quantities. It is required to find that generatrix with the equation $x = x^w(y) \geqslant 0$ which on joining the fixed points i and f, gives the minimum drag

$$\chi = \int_0^{\tau X} (p - p_\infty) dy \tag{1.1}$$

Within the framework of approximate approaches, the solution of this problem is known and is comparatively simple. For instance, in the case of thin bodies ($\tau \ll 1$) when linear theory can be applied at moderate supersonic Mach numbers M_∞, the optimal generatrices are straight lines [1]. If $\tau \leqslant 1$, they are also straight lines [1–3] in the approximation of Newton's law of resistance. However, when $\tau > 1$, the optimal contour in the same approximation consists of an end face $x \equiv 0$ and a straightline segment at an angle of $\pi/4$ to the x-axis. The end face appears as a segment of a boundary extremum due to the constraints on the length ($x \geqslant 0$) and the existence of a boundary inside which Newton's approximation holds, that is, along the contour [2, 3], $dx^w/dy \geqslant 0$. If, for a fixed $M_\infty > 1$, the relative thickness τ is so small that there cannot be a flow around the generatrix if with an attached shock wave ($\tau > \tau^*$ where τ^* is a function of M_∞), then the appearance in the optimal generatrix of an end face is naturally to be expected by virtue of the constraint on the length and also within the framework of the exact gas dynamic equations for an ideal gas, that is, within the framework of Euler's equations. In the axially symmetric case without an end face, an optimal leading edge of fixed length cannot be constructed for any $\tau > 0$ either within the approximation of Newton's (or Busemann's) law of resistance or within the framework of Euler's equations [2–5].

In the approximation of Euler's equations, the first exact result, which refers to the plane leading edges of minimum drag, was established in [6] (also, see [7]) when investigating the flow past bodies which were close to wedge-shaped. In this case, the problem obtained by linearizing the relationships

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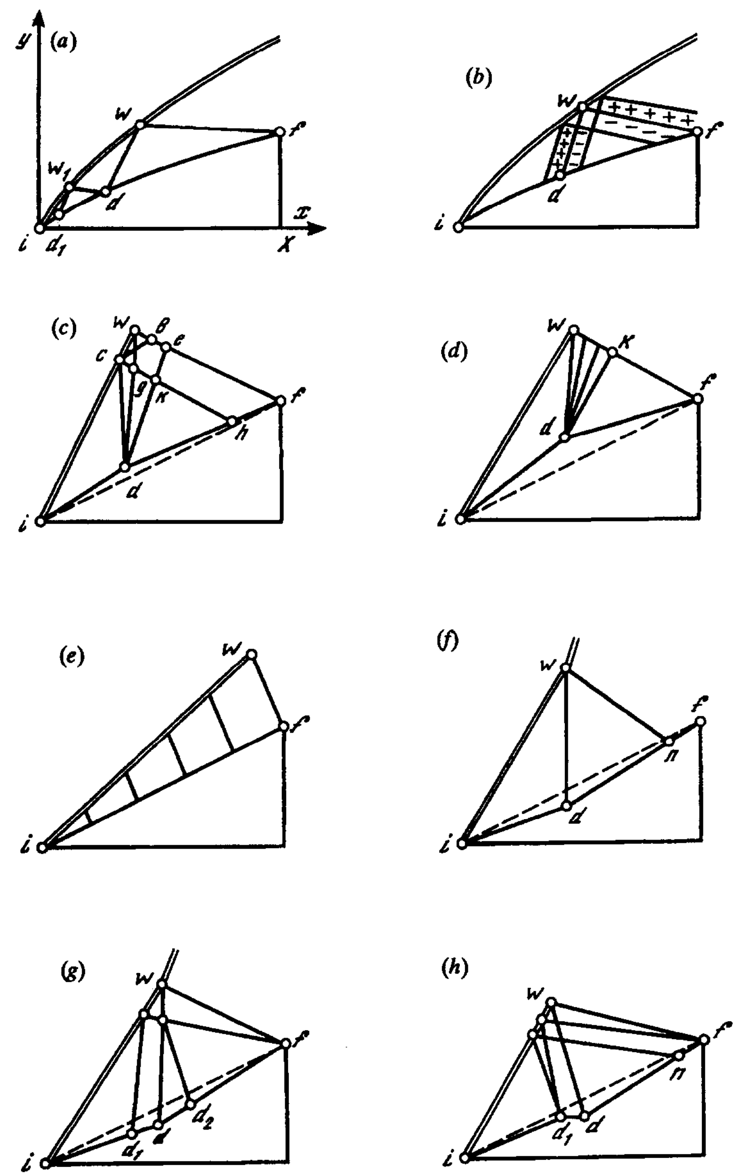

Fig. 1.

in the shock wave, the no-flow condition on the body and the equations of the flow between the slightly distorted generatrices of the body and the shock wave, was considered. Its solution depends, in particular, on the reflection coefficient $\gamma = \delta p^{-}/\delta p^{+}$, where δp^{+} is the pressure perturbation which arrives at the shock wave along the c^{+}-characteristics and δp^{-} is the perturbation in p which leaves from it along the c^{-}-characteristics. Although the theory developed in [6, 7] was linear, the following exact conclusion followed from it: the optimal leading edge is a wedge when $\lambda = 0$. It would appear that the main difference between this conclusion and the analogous result of linear theory or the Newton approximation is the following. The reflection coefficient is a function of M_∞, the angle of inclination of the shock wave σ, the adiabatic index κ in the case of a perfect gas with constant heat capacities, etc. Hence, condition $\lambda(M_\infty, \sigma, \ldots) = 0$ is only satisfied in exceptional cases.

A similar conclusion was arrived at in [8, 9], independently of [6], as the result of an attempt to solve the variational problem which has been formulated above within the framework of Euler's equations and the method of a reference contour. In the latter papers, the curves $\lambda(V_\infty, \sigma) = 0$ were constructed in the $V_\infty\sigma$-plane for a perfect gas with $\kappa = 1.4$ and it was found that, depending on V_∞, this equation (in σ) has from one to three roots. Moreover, it is satisfied in the trivial case when $\sigma = \alpha_\infty$ where α is the Mach angle. This root is of no interest as it corresponds to a "wedge" with $\tau = 0$.

Solution of the linear problem formulated in [6] enabled us to develop the efficient approach of "variation in characteristic ε-bands". This approach, which has been extended to arbitrary bodies (see [4]), has become an important tool for investigating the optimal solutions of variational problems in gas dynamics.

As applied to the problem under consideration when $\lambda \neq 0$, Fig. 1(b) explains the essence of this approach. In it, the contour "which is suspected of being optimal" is only varied in the ε-neighbourhood of the point d. If $\lambda < 0$, the pressure perturbations leaving the body change sign on reflection from the shock wave. The plus and minus signs in Fig. 1(b) correspond to $\delta p > 0$ and $\delta p < 0$. On account of the special choice of the segment of the contour which is varied, after reflection there are only ε-bands with $\delta p < 0$ in it which, as can be shown, leads to an uncompensated reduction in χ. Hence, if $\lambda < 0$ (in the $V_\infty\sigma$-plane, the corresponding point belongs to the subdomain D^- of domain D), a generatrix of the type shown in Fig. 1(b), which does not have a corner point at point d, is not optimal. Furthermore, it follows from this treatment that, when $\lambda < 0$, the optimal contour does have a corner point at the point d around which flow occurs with the formation of a pencil of rarefaction waves. Since, however, there is no corner point when $\lambda_w = 0$, it is natural to expect that its magnitude is proportional to λ_w. Similar reasoning, when applied to the point d_1, indicates that there is also a corner at this point. There is a flow past the corner point in d_1 with the formation of a pencil of rarefaction waves when $\lambda_w, \lambda_{w1} < 0$, and its magnitude is proportional to this product and so on. The reflection coefficient λ is a quantity which is considerably less than unity [6, 7]. It follows from this that the breaks at the points $d_1, \ldots,$ will be much smaller than the "main break" at the point d. For positive λ_w, λ_{w1} and so on, flow occurs around the corresponding corner points with the formation of weak shock waves and this, in turn, leads to a "crowding" of breaks into the neighbourhoods of the corresponding points.

The exceptional complexity of the optimal contours which are obtained by this analysis (see [3]) is compensated by the smallness of the reflection coefficient which has been pointed out above. Their products (especially those including reflection coefficients from the weak shock waves which arise in a flow past "convex" corner points) are so small that only the "main corner point" (or corners) close to the point d turn out to have any substantial effect on the magnitude of χ.

The arguments which have been presented above were already possible at the start of the 1950s, that is, after the publication of [6]. In spite of this, the history of the solution of this problem is extremely complex and, sometimes, also dramatic. An important stage in its solution was the development in [10, 11] of an algorithm for solving the variational problem formulated within the framework "of the general method of Lagrange multipliers". The basis of this algorithm is a numerical calculation by the method of characteristics of the flow past the contour which has been found from the preceding iteration and the solution of the problem for the Lagrange multipliers from the "conjugate" problem which is solved against the flow. The flow parameters and the Lagrange multipliers on the required generatrix, which are found at each iteration, are then substituted into the optimality conditions, the closure of which is used in the procedure for improving the contour which is being profiled. The numerical algorithm which was implemented in [10, 11] for the parameters at the point w which belong to domain D^- of the $V_\infty\sigma$-plane permitted the construction of certain optimal plane leading edges around which a uniform flow of an ideal, perfect gas with $\kappa = 1.4$ occurs. Their contours consist of two practically rectilinear segments which intersect at the point d. The corner at the point d past which flow occurs with the formation of a pencil of rarefaction waves is small while the corners in the neighbourhoods of points $d_1, \ldots$ are not taken into account within the framework of the numerical algorithm in [10, 11].

According to the results in [10, 11] and an analysis within the framework of "variation in characteristic ε-bands" taking into account the smallness of the reflection coefficient λ, the optimal leading edges past which flow occurs with an attached shock wave are close to wedges. If, here, V_∞ and τ belong to the domain D^-, in which the reflection coefficient λ at the point w is negative, the main deviation of an optimal contour from a straight line segment lies in the corner point past which there is a flow with the formation of a pencil of rarefaction waves. These facts enable one, in the class of contours consisting of two rectangular segments which intersect at d, to obtain explicit formulae which determine the practically optimal plane generatrices, to calculate their geometric characteristics and χ, to make a comparison with the results in [10, 11] and to estimate the influence of the factors which have not been taken into account in this approach on the shape of the configurations which have been constructed. This is done below.

2. The rectilinear generatrix $y = x\tau$ is represented by the dashed line in Fig. 1(c), and the oblique shock wave corresponding to it is given by the equation of the straight line $y = x \tan \sigma$. The contour consisting of two rectilinear segments, the shock wave ibw in the corresponding flow, the pencil of rarefaction waves cde, the closing characteristic wf and the streamline bc are represented by the solid lines in the same figure. It follows from the variation in a narrow band described above that a contour with a similar corner point can only be close to the optimal contour in the case of negative λ. Such a conclusion will also be drawn below. However, the sign of λ is arbitrary until it is obtained within the framework of the analysis which is subsequently developed.

Let φ be the value of some parameter for a flow past a wedge and $\varphi + \Delta\varphi$ be its value when the flow occurs past a body with a corner point at d. Let $\Delta\vartheta_-$ and Δp_- be perturbations (increments) of ϑ, that is, of the angle of inclination of the velocity vector to the x-axis and to p on id and in idc, let $\Delta\vartheta_+$ and Δp_+ be perturbations (increments) on dh and in dhk and, finally, let Δp be a perturbation in p on hf. Then, for the increment in χ from (1.1), we find

$$\Delta\chi = \tau X\Delta p_+ + y_d(\Delta p_- - \Delta p_+) + (\Delta p_- - \Delta p_+)\Delta y_d + \int_h^f (\Delta p - \Delta p_+)dy \tag{2.1}$$

where $y_d + \Delta y_d$ is the ordinate of the point d of the contour with a corner point and y_d is the corresponding magnitude for the wedge, which is equal to

$$y_d = \tau X \frac{\sin(\vartheta + \alpha - \sigma)}{\sin(\sigma - \vartheta + \alpha)}$$

Since X and τ are fixed, the required contour idf is determined by two increments, $\Delta\vartheta_-$ and $\Delta\vartheta_+$, $\Delta\vartheta_-$ and Δx_d, etc. for example. In addition, the latter must satisfy the condition that one of the c^+-characteristics of the pencil must pass into the point w, that is, the point of intersection of the "perturbed" shock wave and the closing c^--characteristic. The formulation of this condition is somewhat simplified if $\Delta\vartheta$, the increment in ϑ on dg, and $\Delta\sigma$, the increment in σ on the rectilinear segment of the shock wave ic, are taken as the required increments. In this case, to determine the optimal values of $\Delta\vartheta^m$ and $\Delta\sigma^m$ which give a minimum in $\Delta\chi$, it is necessary to represent the right-hand side of (2.1) with an accuracy of no less than to quadratic terms in $\Delta\vartheta$ and $\Delta\sigma$, inclusive. The linear approximation formulae [5, 6] for determining $\Delta\vartheta^m$ and $\Delta\sigma^m$ are obviously insufficient, with the exception of the special cases $\Delta\vartheta^m = \Delta\sigma^m = 0$, when a wedge is optimal.

By virtue of the conditions on an oblique shock wave, the increment in any parameter $\Delta\varphi_-$ on id and in idc is a known function of $\Delta\sigma$ and, moreover, according to what has been said above and formula (2.1), it is necessary to retain the linear and quadratic terms in $\Delta\sigma$, that is, to put

$$\Delta\varphi_- = a_\varphi(1 + b_\varphi\Delta\sigma)\Delta\sigma \tag{2.2}$$

For a specified free stream, the coefficients a_φ and b_φ in (2.2) are known functions of σ or ϑ. Numerical differentiation may be used to determine them by calculating the three values of φ: $\varphi(\sigma)$, $\varphi(\sigma + \delta\sigma)$ and $\varphi(\sigma - \delta\sigma)$ with $\delta\sigma \ll \sigma$ using the relations in the oblique shock wave for each σ. In this investigation, the coefficients were found both by numerical differentiation and using explicit formulae as a check. In the case of a perfect gas, the formulae for the coefficients a_φ and b_φ which are subsequently used have the form

$$a_p = \frac{2\sin 2\sigma}{\kappa+1}, \quad b_p = \operatorname{ctg} 2\sigma, \quad a_\vartheta = 1 + b_1 - a_1$$

$$a_1 = \frac{(\kappa-1)\cos^2(\sigma-\vartheta)}{(\kappa+1)\cos^2\sigma}, \quad b_1 = \frac{8\operatorname{ctg}2\sigma\cos^2(\sigma-\vartheta)}{(\kappa+1)M_\infty^2\sin 2\sigma}$$

$$b_\vartheta = \frac{1}{a_\vartheta}\left[(b_1 - a_1)^2\operatorname{tg}(\sigma-\vartheta) - a_1\operatorname{tg}\sigma - b_1\frac{3+\cos 4\sigma}{\sin 4\sigma}\right]$$

$$a_\rho = \frac{2(1-\varepsilon^2)\kappa M_\infty^2\sin 2\sigma}{(\kappa+1)(1+\varepsilon\kappa M_\infty^2 p)^2}, \quad \varepsilon = \frac{\kappa-1}{\kappa+1}, \quad a_V = \frac{\kappa(p a_\rho - \rho a_p)}{(\kappa-1)\rho^2 V}$$

$$a_a = \frac{\kappa(\rho a_p - p a_\rho)}{2\rho^2 a}, \quad a_M = \frac{a_V - M a_a}{a}, \quad a_\alpha = \frac{-a_M}{M\sqrt{M^2-1}}$$

$$a_S = \frac{\kappa p a_\rho - \rho a_p}{\kappa p \rho}S, \quad S = \rho p^{-1/\kappa}$$

On the right-hand sides of these formulae there are the following parameters beyond the oblique shock wave: the modulus of the velocity V and the velocity of sound $a = \sqrt{(\kappa p/\rho)}$ divided by V_∞°. The Mach number

$m = V/a$, the Mach angle α, and the angle of rotation of the flow in the shock wave ϑ, which is equal to the angle of inclination of the velocity vector to the x-axis, and also the entropy function S, the density and the pressure divided by $(\rho_\infty^\circ V_\infty^{\circ 2})^{1/\kappa}/\rho_\infty^\circ$, ρ_∞° and $\rho_\infty^\circ V_\infty^{\circ 2}$, respectively, which are functions of M_∞, σ and κ that are determined using known relations.

The equations [12]

$$d\Delta\vartheta \pm A(p+\Delta p, S+\Delta S)d\Delta p = 0, \qquad A(p,S) = \sqrt{M^2-1}\,/\,(\rho V^2) \tag{2.3}$$

are satisfied along the c^+- and c^--characteristics.

Henceforth, plus (minus) signs correspond to $c^+(c^-)$-characteristics. To terms in Δ^2, inclusive, where Δ is a small parameter which characterizes the magnitude of $\Delta\sigma$, $\Delta\vartheta$ or $\Delta\vartheta_d \equiv |\Delta\vartheta_+ - \Delta\vartheta_-|$, we have from this that

$$\begin{gathered} d\Delta\vartheta \pm A(1 + C_p\Delta p + C_S\Delta S)d\Delta p = 0 \\ C_p = \frac{4(M^2-1)-(\kappa+1)M^4}{2\kappa M^2(M^2-1)p}, \quad C_S = \frac{4+2(\kappa-2)M^2-(\kappa-1)M^4}{2(\kappa-1)M^2(M^2-1)S} \end{gathered} \tag{2.4}$$

The coefficients C_p and C_s which are written out in (2.4) for a perfect gas, as well as a_φ and b_φ, can also be found by numerical differentiation (as was done as a check) using the values of $A(p \pm \delta p, S)$ and $A(p, S \pm \delta S)$ with $\delta p \leqslant p$ and $\delta s \leqslant S$, respectively.

After integration over the characteristic ε-bands in the neighbourhood of the point of reflection from the shock wave (Fig. 1b), the equations of the characteristics (2.3) or (2.4) (written in the linear approximation to terms in Δ, inclusive) lead to a very simple formula for the reflection coefficient λ. By using the results of this integration (the equation for the c^+-characteristic is integrated over the c^--band and vice versa), performing some simple calculations and taking into account the definition of ap and a_ϑ in (2.2), we find that

$$\lambda = (Aa_p - a_\vartheta)/(Aa_p + a_\vartheta) \tag{2.5}$$

In the case of shock waves of the weak family when $M \geqslant 1$, both the pressure and the angle ϑ become larger as σ increases. Hence, in the cases under consideration, a_p and a_ϑ as well as A are positive and only the numerator in (2.5) can vanish. According to (2.5) $|\lambda| < 1$, which has been noted, for example, in [6, 7]. Finally, it follows from the expressions for a_p and a_ϑ presented earlier that $\lambda = 0$ when $\vartheta = 0$.

According to what has been said above, Δp_- and Δ_{p+} in the first two terms on the right-hand side of (2.1) must be determined to terms in Δ^2, inclusive. On using formula (2.2) for this, integrating (2.4) along any c^--characteristics from the shock wave up to the wall and taking account of the fact that ΔS is constant under the streamline bc and is determined by the same formula (2.2), we obtain

$$\begin{aligned} \Delta p_+ &= \Delta p_- + \frac{1}{A}\{\Delta\vartheta_+ - a_\vartheta(1+b_\vartheta\Delta\sigma)\Delta\sigma + (a_\vartheta\Delta\sigma - \Delta\vartheta_+)\times \\ &\times\left[\frac{C_p}{2A}(\Delta\vartheta_+ - a_\vartheta\Delta\sigma) + (a_pC_p + a_SC_S)\Delta\sigma\right]\Bigg\} \\ \Delta p_- &= a_p(1+b_p\Delta\sigma)\Delta\sigma \end{aligned} \tag{2.6}$$

If the subscript is removed in the first formula of (2.6) and from Δp and $\Delta\vartheta_+$, then one obtains an expression which yields Δp on dg in terms of $\Delta\sigma$ and $\Delta\vartheta$ up to terms in Δ^2.

The condition that a contour with a corner point should join points i and f with an accuracy up to terms in Δ^2, inclusive, reduces to the equation

$$\begin{gathered} x_d(1+\Delta\vartheta_-\operatorname{tg}\vartheta)\Delta\vartheta_- + (X-x_d)(1+\Delta\vartheta_+\operatorname{tg}\vartheta)\Delta\vartheta_+ + \Delta x_d(\Delta\vartheta_- - \Delta\vartheta_+) = 0, \\ x_d = X\frac{\sin(\vartheta+\alpha-\sigma)}{\sin(\sigma-\vartheta+\alpha)} \end{gathered}$$

Using this and expression $\Delta\vartheta_+$ with the same accuracy, as (2.26) contains $\Delta\vartheta_+$ to the first and second powers, we obtain

$$\Delta\vartheta_+ - \Delta\vartheta_- = \Theta_\sigma\Delta\sigma + \Theta_{\sigma\sigma}(\Delta\sigma)^2 + \Theta_{\sigma x}\Delta\sigma\,\Delta x_d \tag{2.7}$$

$$\Theta_\sigma = -a_\vartheta\frac{\sin(\sigma-\vartheta+\alpha)}{2\sin(\sigma-\vartheta)\cos\alpha}, \quad \Theta_{\sigma x} = -\frac{a_\vartheta\sin^2(\sigma-\vartheta+\alpha)}{4X\sin^2(\sigma-\vartheta)\cos^2\alpha}$$

$$\Theta_{\sigma\sigma} = b_\vartheta\Theta_\sigma - a_\vartheta^2\operatorname{tg}\vartheta\frac{\sin(\vartheta+\alpha-\sigma)\sin(\sigma-\vartheta+\alpha)}{4\sin^2(\sigma-\vartheta)\cos^2\alpha}$$

We substitute $\Delta\vartheta_+$ from (2.7) into (2.6) and retain terms up to Δ^2, inclusive, keeping in view their subsequent use in (2.1). As a result, we arrive at the expression

$$\Delta p_+ = \Delta p_- + P_\sigma\Delta\sigma + P_{\sigma\sigma}(\Delta\sigma)^2 + P_{\sigma x}\Delta\sigma\,\Delta x_d \tag{2.8}$$

$$P_\sigma = \frac{\Theta_\sigma}{A}, \quad P_{\sigma\sigma} = \frac{\Theta_{\sigma\sigma}}{A} - \frac{\Theta_\sigma}{A}\left(\frac{C_p\Theta_\sigma}{2A} + a_pC_p + a_SC_S\right), \quad P_{\sigma x} = \frac{\Theta_{\sigma x}}{A}$$

In (2.1), Δy_d is multiplied by $\Delta p_- - \Delta p_+ = O(\Delta)$. Hence, confining ourselves solely to the linear part in the expression for Δy_d in terms of $\Delta\sigma$ and Δx_d, we find that

$$\Delta y_d = \Delta x_d\operatorname{tg}\vartheta + \frac{a_\vartheta X\sin(\vartheta+\alpha-\sigma)}{\cos^2\vartheta\sin(\sigma-\vartheta+\alpha)}\Delta\sigma \tag{2.9}$$

If (2.8) with Δp_- from (2.6) and (2.9) is substituted into the terms outside the integral on the right-hand side of (2.1), it then takes the form

$$\Delta\chi = \chi_\sigma\Delta\sigma + \chi_{\sigma\sigma}(\Delta\sigma)^2 + \int_h^f(\Delta p - \Delta p_+)dy \tag{2.10}$$

$$\chi_\sigma = \frac{\lambda X}{A}(Aa_p + a_\vartheta)\operatorname{tg}\vartheta, \quad \chi_{\sigma\sigma} = X\left[a_pb_p\operatorname{tg}\vartheta + 2P_{\sigma\sigma}\frac{\sin(\sigma-\vartheta)\cos\alpha}{\sin(\sigma-\vartheta+\alpha)}\operatorname{tg}\vartheta + \right.$$

$$\left. + \frac{a_\vartheta^2\sin(\vartheta+\alpha-\sigma)}{2A\cos^2\vartheta\sin(\sigma-\vartheta)\cos\alpha}\right]$$

It is clear that χ_σ vanishes simultaneously with the reflection coefficient and is also independent of the magnitude of λ in the case of a shock wave which has degenerated into a characteristic ($\vartheta = 0$). Since, in the latter case according to (2.5) and the formulae for a_p and a_ϑ, $\lambda = 0$ simultaneously, the coefficient χ_σ on the V_∞ axis of the $V_\infty\tau$-plane vanishes with a higher order than on the other lines of D^0 of this plane, were $\lambda(V_\infty, \tau) = 0$.

After substituting (2.8) and (2.9) into (2.1), the terms $\Delta\sigma\Delta x_d$ occurring in (2.1) and (2.8) cancel out, which simplifies the determination of the optimal $\Delta\sigma$ and $\Delta\vartheta$. Since this product does not occur in (2.10), it now remains to find the integral hf occurring in (2.10) up to terms in Δ^2. The difference $y_f - y_h = O(\Delta)$. Hence, in deriving the required formula, it is sufficient to approximate the change in the integrand from zero when $y = y_h$ up to $\Delta p_f - \Delta p$ when $y = y_f$ with a straight line. As a result, we obtain

$$\int_h^f(\Delta p - \Delta p_+)dy = \frac{1}{2}(\Delta p_f - \Delta p_+)(y_f - y_h) \tag{2.11}$$

We find the difference $y_f = y_h$ as the width of the characteristic band which is obtained due to reflection from the shock wave in the part cdw of the pencil of rarefaction waves. All the c^+-characteristics of this pencil, which are only distorted in a small neighbourhood of the shock wave (to the right of the c^--characteristic ck), diverge as a fan from the point d. In dch, the flow is of the simple wave type. Consequently, here

$$\Delta\alpha = \Delta\alpha_- + \alpha_\vartheta(\Delta\vartheta - \Delta\vartheta_-) = (a_\alpha - a_\vartheta\alpha_\vartheta)\Delta\sigma + \alpha_\vartheta\Delta\vartheta \tag{2.12}$$

with an accuracy to terms in Δ, inclusive.

The derivative α_ϑ can be found by differentiating the formulae describing the simple wave. This is simpler to do numerically, by calculating $\alpha(\sigma, \vartheta \pm \delta\vartheta)$ with $\delta\vartheta \ll \vartheta$ using them. This method is used below.

The c^--characteristic *ch* and *wf* reflected from the shock wave as well as the c^+-characteristics of the pencil are rectilinear almost everywhere. The exceptions are the neighbourhoods of the point *c* for *ch* and of the points *w* and *f* in the case of *wf*. In the reflected characteristic band outside the neighbourhoods, a simple wave type flow is obtained with rectilinear c^--characteristics which diverge when $\lambda > 0$ and converge when $\lambda < 0$. The change in the width of the band associated with this is proportional to $\lambda(\Delta\vartheta_- - \Delta\vartheta)$. We now take account of the facts which have been noted, the equalities (2.2) and (2.12) and the compatibility conditions (2.4), while retaining the necessary number of terms everywhere. As a result, we find, with the required accuracy, that

$$\Delta p_f - \Delta p_+ = 2\lambda(\Delta\vartheta - \Delta\vartheta_-)/A, \quad y_f - y_h = \zeta(\Delta\vartheta_- - \Delta\vartheta) \tag{2.13}$$

$$\zeta = X\left[1 + \lambda\frac{\sin(\vartheta + \alpha - \sigma)}{\sin(\sigma - \vartheta + \alpha)}\right]\frac{(1 + \alpha_\vartheta)\operatorname{tg}\vartheta\sin(\sigma - \vartheta)}{\sin\alpha\sin(\vartheta + \alpha - \sigma)}$$

The difference between the expression in the square brackets in the formula for ζ and unity characterizes the broadening or narrowing of the characteristic band. This expression is always positive. When $\lambda < 0$, this is confirmed when account is taken of the fact that $|\lambda| < 1$. On substituting (2.13) into (2.11) and the result of this into (2.10), we arrive at the required expression

$$\Delta\chi = \chi_\sigma\Delta\sigma + \chi_{\sigma\sigma}(\Delta\sigma)^2 + \chi^f(\Delta\vartheta - \Delta\vartheta_-)^2, \quad \chi^f = -\lambda\zeta/A \tag{2.14}$$

For the independent geometric parameters in (2.14), it is more convenient to take $\Delta\sigma$ and the difference $\Delta\vartheta - \Delta\vartheta_-$ rather than $\Delta\sigma$ and $\Delta\vartheta$ with the replacement of $\Delta\vartheta_-$ by $\alpha_\vartheta\Delta\sigma$. In this case the necessary and sufficient conditions for a minimum of $\Delta\chi$ (the equality of the first derivatives to zero and the positiveness of the second derivatives at once) give

$$\begin{gathered}\Delta\sigma^m = -\chi_\sigma/(2\chi_{\sigma\sigma}), \quad \Delta\vartheta^m = \Delta\vartheta_-^m = a_\vartheta(1 + b_\vartheta\Delta\sigma^m)\Delta\sigma^m \\ \chi_{\sigma\sigma} > 0, \quad \chi^f > 0\end{gathered} \tag{2.15}$$

where the superscript *m* denotes values which give a minimum value of χ.

Only λ can be negative in the formula for χ^f. Actually, the positiveness of the square bracket has been written about above. The non-negativeness of the trigonometric functions occurring in χ^f and the coefficient A is obvious. The sum $1 + \alpha_\vartheta$ is also positive since a reduction in α corresponds to a reduction in ϑ accompanying flow past a corner point. Hence, according to the second inequality from (2.15), the scheme of Fig. 1(c) can only be optimal when $\lambda < 0$, that is, in the domain D^- of the $V_\infty\tau$-plane. This conclusion (but not the formulae for determining $\Delta\sigma^m$ and $\Delta\vartheta^m$) are identical with the assertion made within the framework of "variation in the ε-band".

Calculations for a perfect gas with different κ showed that $\chi_{\sigma\sigma} > 0$ everywhere in D. According to (2.10), the signs of χ_σ and λ are the same. Hence, by virtue of the first equality of (2.15), $\Delta\sigma^m > 0$ when $\lambda < 0$ and $M \neq 1$ as it must be in the case of optimal contours of the type under investigation. The meaning of the second condition of (2.15) is also understandable: the position of the main corner point of the optimal contour when $\lambda < 0$ is such that the initial c^+-characteristic of the pencil of rarefaction waves (Fig. 1d) arrives at the point *w* of intersection of the shock wave and the closing c^--characteristic of *wf*. As a result, the rarefaction waves which are reflected from the shock wave as compression waves are not incident on the end segment of the generatrix and do not increase χ. Optimal contours, constructed using the iterative procedure developed in [10, 11] using the general method of Lagrange multipliers and the method of characteristics, do not possess this property. Despite this as is shown below, they give only a slightly greater advantage, compared with a wedge, than the almost optimal generatrices of this work. The reason for this difference and its order of magnitude are explained in the following section.

According to (2.14) and (2.15), the magnitude of χ for the contour which is optimal in the approximation under consideration is less than the magnitude of χ for the rectilinear generatrix, that is, for the wedge, by

$$\Delta\chi^m = -\chi_\sigma^2/(4\chi_{\sigma\sigma}) \tag{2.16}$$

The stipulation made above concerning $M \neq 1$ is associated with the unlimited increase in the coefficients

χ_σ and $\chi_{\sigma\sigma}$ when $M \to 1$. By virtue of the formulae which have been presented above, for values of M close to unity $\chi_\sigma \sim (M-1)^{-1/2}$ and $\chi_{\sigma\sigma} \sim (M-1)^{-5/2}$. Hence, from (2.15) and (2.16), when $M = 1$, we have that

$$\Delta\sigma^m \sim \lambda(M-1)^2, \quad \Delta\chi^m \sim \lambda^2(M-1)^{3/2}$$

Consequently, $\Delta\sigma^m = 0$ when $M = 1$, that is, a wedge is the body of minimum drag not only if $\lambda = 0$ but also on the boundary of the domain D, which corresponds to a sonic velocity beyond the shock wave and $\lambda \neq 0$. This result is so unexpected that it requires additional explanation. In the case of sonic flow (with respect to the Mach number beyond the shock wave) past a wedge (Fig. 1e), the c^+- and c^--characteristics coincide and are perpendicular to its surface. Hence, even within the framework of a purely linear approach which, as a matter of fact, is inapplicable when $M = 1$, variation in the characteristic band, unlike the situation depicted in Fig. 1(b), does not "reject" a rectilinear generatrix when $\lambda \neq 1$ in the case of Fig. 1(e).

Yet another explanation of the optimality of a sonic wedge is obtained by considering the evolution of Fig. 1(a) when $\delta = M - 1 \to 0$. Let $\varepsilon > 0$, let $n > 1$ be fixed and let d_n be the corresponding point of "reflection". Then, for any $n < \infty$ and $\varepsilon > 0$, it is possible to choose $\delta = \delta(n, \varepsilon)$ such that all points d_k with $k \leqslant n$ lie in the ε-neighbourhood of the point f and, when $\delta = 0$, they merge with it. Figure 1(e) also explains the reason for the inapplicability of the "control contour method" in this case. A mandatory condition for this method to be applicable [3] is the presence of two segments in the required generatrix: an initial segment which determines the shape of the shock wave iw and a final segment which determines the distribution of the parameters in the closing characteristic wf. In Fig. 1(b), these segments are id and df. In the case of Fig. 1(e), the segment df degenerates to the point f.

By virtue of [6, 7] in the case of a generatrix if which is close to a straight line, the main term $\delta\chi$ of the increment ("variation") in χ is given by the formula

$$\delta\chi = \sum_{n=0}^{\infty} a_n \delta x_{dn}, \quad a_n \sim \lambda^{n+1}$$

in which δx_{dm} is the variation (incremental growth for fixed y) in x at the point d_n and $d_0 \equiv d$. It has already been pointed out that it was proposed that $M > 1$ in [6, 7] when deriving this formula. On the other hand, according to [3], the formula for the first variation in χ, which is obtained by the "general method of Lagrange multipliers" (MLM), reduces to the same expression, but now also for a sonic wedge ($M = 1$) when there is a small variation in its generatrix. Since, when $M = 1$, the set of points d_n with $\delta x_{dn} \neq 0$ is empty, a sonic wedge satisfies the necessary condition for an extremum of χ within the framework of MLM. Finally, in MLM, the above-mentioned "small variation" of the generatrix if must ensure the smallness of the variations of all the parameters. When $M = 1$, this imposes extremely rigorous constraints on the smoothness of the contour which is varied. An arbitrary variation of if, which causes the breakdown of the sonic flow, creates problems with the application of any of the optimization methods which assume that the variations are small. For instance, if finite positive $\Delta\sigma^m$ were to be obtained from (2.15) when $M \to 1$, it would be impossible to use these results. Conversely, $\Delta\sigma^m = 0$ when $M = 1$ is evidence of the compatibility of the result obtained and the method of finding it.

In the approach which has been developed, the calculation of $\Delta\chi^m$ and the reduction in the drag coefficient (in percent)

$$\delta c_x = \frac{\Delta c_x}{c_x} \times 100 = -\frac{\Delta\chi^m}{\chi} \times 100 \tag{2.17}$$

do not require a knowledge of the coordinates of the corner point d. In the case of a known flow system (Fig. 1d) with $\Delta\sigma^m$ and $\Delta\vartheta^m$ from (2.15), it is easy to construct a numerical procedure for calculating its coordinates. Since, however, Δx_d in (2.1) and (2.9) only occurs multiplied by a quantity of the order of magnitude of Δ, and (2.10) does not contain Δx_d at all, it is sufficient to find it to the first order of magnitude.

The basic scheme for obtaining a linear relationship between Δx_d and $\Delta\sigma^m$ for the optimal generatrix is as follows. Initially, Δx_w and Δy_w are expressed in terms of $\Delta\sigma^m$ and Δx_d from a consideration of the rectilinear segment of the shock wave iw and the rectilinear characteristic dw. Here, the equality (2.9) is used and only terms which are linear with respect to $\Delta\sigma^m$ and Δx_d are retained in the final formulae. On the other hand, $\Delta x_f = \Delta y_f = 0$ and the points f and w are joined by a characteristic which is only warped close to the points w and f. This yields a linear relationship between Δx_w, Δy_w, $\Delta\vartheta_+$ and $\Delta S = a_S\Delta\sigma^m$. We now substitute $\Delta\vartheta_+$, determined by the linear part of (2.7) with $\Delta\vartheta_- = a_\vartheta\Delta\sigma^m$ and the expressions for Δx_w and Δy_w in terms of $\Delta\sigma^m$ and $4\Delta x_d$, found up to this point, into it. Finally, we obtain

$$\Delta x_d = x_\sigma \Delta\sigma^m \tag{2.18}$$

$$x_\sigma = X\{a_\vartheta[\sin 2\alpha \sin(\sigma-\vartheta+\alpha)\cos(\sigma-2\vartheta-\alpha)+\cos\vartheta\sin^2(\vartheta+\alpha-\sigma)]+$$
$$+\ a_\alpha \cos\vartheta \sin 2\alpha \sin 2(\sigma-\vartheta)-a_\vartheta\alpha_\vartheta \cos\vartheta\sin(\vartheta+\alpha-\sigma)\sin(\sigma-\vartheta+\alpha)-\cos\vartheta\sin^2 2\alpha\}\times$$
$$\times\ [\cos\vartheta\sin 2\alpha\sin^2(\sigma-\vartheta+\alpha)]^{-1}$$

with the required accuracy.

3. In that part D^+ of domain D of the $V_\infty\tau$-plane where $\lambda > 0$, contours with a convex main corner point cannot be close to optimal. Instead of these, it is natural to investigate configurations with a corner point past which a flow occurs with the formation of a weak shock wave dw. When $\lambda > 0$, it is also reflected from the leading shock wave iw by the weak shock wave as shown in Fig. 1(f). In this figure and subsequently, weak shock waves are depicted by bold lines. If the pressure drop in dw is equal to $[p]^+ \equiv \Delta p_+ - \Delta p_-$, then, for the reflected shock wave, $[p]^- = \lambda[p]^+$ and, in the case of its reflection from the wall df beyond the shock wave (in hf),

$$\Delta p - \Delta p_+ = 2\lambda[p]^+ = 2\lambda(\Delta\vartheta_+ - \Delta\vartheta_-)/A > 0 \tag{3.1}$$

Here, account has been taken of the fact that, in weak shock waves and in simple waves ("of the same family": in the case of Fig. 1f, it is in a simple wave with rectilinear c^+-characteristics) the relations between the increments in p and ϑ are distinguished by quantities of the order of Δ^3. On account of this, in particular, the relationship between $\Delta p_+ - \Delta p_-$ and $\Delta\vartheta_+ - \Delta\vartheta_-$ is retained as well as expression (2.10) for $\Delta\chi$ with the previous coefficients χ_σ and $\chi_{\sigma\sigma}$. Now, however, when account is taken of (3.1), we have

$$\Delta\chi = \chi_\sigma\Delta\sigma + \chi_{\sigma\sigma}(\Delta\sigma)^2 + 2\lambda(\Delta\vartheta_+ - \Delta\vartheta_-)\Delta y_{fh}/A =$$
$$= \chi_\sigma\Delta\sigma + \chi_{\sigma\sigma}(\Delta\sigma)^2 + 2\lambda\Theta_\sigma\Delta\sigma\Delta y_{fh}/A \tag{3.2}$$

instead of (2.11).

The difference $\Delta y_{fh} = y_f - y_h$, as previously, is a quantity of the order of Δ and the second expression is obtained from the first by virtue of the fact that formula (2.7) holds for any small breaks. Since, according to this formula, $\Theta_\sigma < 0$ and $\Delta y_{fh} \geqslant 0$ then, in the case of the generatrices which are now being investigated with $\Delta\sigma < 0$, the last term in (3.2) is either positive (when $y_h < y_f$) or equal to zero. The second possibility occurs if the weak shock wave reflected from iw does not arrive in the segment df or, in the limiting case, arrives at its terminal point f. A contour, which is close to the optimal contour, corresponds, in fact, to this limiting situation (Fig. 1g).

The following fact is important for the proof of this assertion: if the position of point d is such that the reflected weak shock wave is not incident on df then, in (3.2) as well as in (2.11), there are no terms of the order of Δ and Δ^2 which contain Δx_d. On account of this, for a fixed $\Delta\vartheta_-$ or, what is the same, a fixed $\Delta\sigma$, a small displacement of the point d along id changes $\Delta\chi$ by larger orders of magnitude. On carrying out the necessary operations, we obtain

$$\Delta\chi = \chi_\sigma\Delta\sigma + \chi_{\sigma\sigma}(\Delta\sigma)^2 + \chi_{\sigma\sigma x}(\Delta\sigma)^2\Delta x_d \tag{3.3}$$

$$\chi_{\sigma\sigma x} = \frac{a_\vartheta^2\sin(\sigma-\vartheta+\alpha)}{4A\sin^2(\sigma-\vartheta)\cos^2\vartheta\cos^2\alpha}\left[\sin(\vartheta+\alpha-\sigma)-\left(\frac{C_p}{2A}\cos\vartheta+\sin\vartheta\right)\sin\vartheta\sin(\sigma-\vartheta+\alpha)\right]$$

instead of (3.2).

In the case of a perfect gas, at least, the coefficient $\chi_{\sigma\sigma x}$ is positive not only in D^+ but also everywhere in D. In the case of a fixed $\Delta\sigma$, a reduction in Δx_d therefore reduces $\Delta\chi$. A displacement of points d and w and, together with them, the reflected weak shock wave to the left corresponds to a decrease in Δx_d. For a certain Δx_d, this shock wave arrives at the point f. When Δx_d is reduced further the difference $y_f - y_h \equiv \Delta y_{fh}$ becomes positive. After this, it is necessary to add into (3.3) the last term from (3.2). For fixed $\Delta\sigma$, it can be shown that the change in the increment Δx_d which corresponds to $\Delta y_{fh} > 0$ is equal to $\delta\Delta x_d = (\delta\Delta y_d)\mathrm{ctg}\,\vartheta = -\Delta y_{fh}\,\mathrm{ctg}\,\vartheta$ and the change in the increment $\Delta\chi$ associated with this is given by the formula

$$\delta\Delta\chi = \chi_y\Delta y_{fh},\quad \chi_y = \Delta\sigma'''(2\lambda\Theta_\sigma - A\chi_{\sigma\sigma x}\Delta\sigma'''\,\mathrm{ctg}\,\vartheta)/(2A) \tag{3.4}$$

with $\Delta\sigma'''$ from (2.15).

According to calculations which were carried out, the factor χ_y in D^+ is positive, which also proves the assertion made above concerning the closeness of a contour with a reflected shock wave arriving at the point f to the optimal contour. Moreover, the factor χ, which is non-negative everywhere in D, vanishes only on D^0, that is, on lines where the reflection coefficient is equal to zero. The positiveness of χ, not only in D^+ but also in D^- is natural. Actually, if the flow around a convex corner point is considered in the linear approximation, the pencil of rarefaction waves in Fig. 1(c) and (d) is replaced by a weak rarefaction shock wave which is reflected from the leading shock wave by the weak shock wave (in D^-, the reflection coefficient $\lambda < 0$). As a result, we again arrive at expression (3.4) for $\delta\Delta\chi$ and the optimality condition for the generatrix shown in Fig. 1(d) reduces to the inequality $\chi_y > 0$.

In the case under consideration, as when $\lambda < 0$, the maximum reduction in the drag is determined using (2.17) with $\Delta\chi^m$ from (2.16). Here, as before, there is no need to know the magnitude of Δx_d which, for $\lambda > 0$, is given by the formula

$$\Delta x_d = X\left\{\frac{a_\alpha \sin 2(\sigma-\vartheta - \sin 2\alpha}{\sin^2(\sigma-\vartheta+\alpha)} + \frac{a_\vartheta(1+\alpha_\vartheta)}{2\sin 2\alpha}\left[(2+\lambda)\frac{\sin(\sigma-\vartheta-\alpha)}{\sin(\sigma-\vartheta+\alpha)} - 1\right] + \right.$$

$$+ \frac{a_\vartheta \sin\alpha}{\cos\vartheta\cos(\vartheta+\alpha)}\left[\frac{\sin^2(\sigma-\vartheta-\alpha)}{\sin^2(\sigma-\vartheta+\alpha)} + \frac{\sin 2\vartheta \sin(\sigma-\vartheta-\alpha)}{\sin 2\alpha \sin(\sigma-\vartheta+\alpha)} + \right.$$

$$\left.\left. + 2\frac{\cos(\sigma-2\vartheta-\alpha)}{\sin^2(\sigma-\vartheta+\alpha)}\cos\alpha\cos\sigma\right]\right\}\Delta\sigma''' \tag{3.5}$$

When account is taken of the difference in the flow schemes in Fig. 1(g) and (d), the derivation of formula (3.5) is similar to the derivation of (2.18). The principal difference in the derivation of (3.5) is associated with the fact that the weak shock wave dw, which, here, replaces a pencil of characteristics, goes along the bisector of the c^+-characteristics up to and after it and the weak shock wave wf goes along the bisector of the analogous c^--characteristics.

4. The formulae presented above give generatrices which are close to optimal leading edges with a single "main" corner point past which flow occurs with the formation of either a pencil of rarefaction waves (in D^-, that is, when $\lambda < 0$) or a weak shock wave (in D^+, that is, when $\lambda > 0$). The calculation of thousands of such generatrices requires several minutes using a 486 AT personal computer. As a result, it was possible to construct the isolines of any of their local and integral characteristics for all velocities V_∞ of the supersonic free stream ($M_\infty > 1$) divided by its critical velocity and relative thickness τ corresponding to the flow past the required generatrices with an attached shock wave.

As an illustration of what has been said, the isolines of δc_x, $\Delta\vartheta_-$ and $\delta\vartheta_d \equiv |\Delta\vartheta_+ - \Delta\vartheta_-|$ in the $W_\infty\tau$-plane, constructed using 6400 points, are shown in Figs 2–4(a) for a perfect gas with $\kappa = 1.4$. Here, $W_\infty = (V_\infty - 1)/(V_\infty{}^m - 1)$ with a maximum velocity $V_\infty{}^m = \sqrt{((\kappa+1)/(\kappa-1))} \simeq 2.45$. The zero level lines are the W_∞ axis, the two isolines entering this axis at finite angles and the dashed "sonic" line, that is, the boundary of domain D which corresponds to sonic wedges. According to what has been said previously, the results referring to these isolines and, in particular, their shape in the $W_\infty\tau$-plane, are exact. We shall refer to "non-trivial" lines, that is, zero level lines which differ from the W_∞ axis, as "null" lines. As W_∞ increases the left null isoline approaches the "sonic" line on which $\lambda < 0$ and, in the scale of Figs 2–4, merges with it. In the domain D^+, that is, in the strip between the null isolines, the reflection coefficient is positive but it is negative outside D^+.

Values of δc_x, that is, the relative advantages (in percent) with respect to the drag, are given in Fig. 2(a) close to the isolines. Moreover, henceforth, the quantities $\Delta\chi^m$ for the generatrices constructed were determined not using the approximate formula (2.16) but using the exact relationships which describe oblique shock waves and a centred rarefaction wave. The geometric parameters required for this computation were successively calculated as follows: $\Delta\sigma^m$ and $\Delta\vartheta^m{}_-$ using (2.15), Δx_d using (2.18) in D^- and using (3.5) in D^+, $\Delta\vartheta_+$ using (2.7) and, finally, Δy_d using (2.9). The numbers on the isolines in Fig. 3(a) and Fig. 4(a) are the values of $\Delta\vartheta_-$ multiplied by 100 and the values of the deflection angle $\Delta\vartheta_d$ (in radians).

The deflection angles and the advantages with respect to c_x of the leading edges in this paper which are close to optimal, and the families of contours constructed in [10, 11] using an iterative procedure based on the general method of Lagrange multipliers are compared in Table 1 (quantities from [10, 11] are without the superscript m). The relative difference $\Delta\delta c_x = (\delta c_x - \delta c_x^m)/\delta c_x$ in the advantages is shown in the final column of this table. As a rule (the exception is the first version, where $\Delta c_x < 0.02\%$), it does not exceed 0.1 with a considerably greater (from 10 to 38%) difference in the deflection angles. The insignificant excess (by 2% of δc_x^m over δc_x in the fourth version is due to the insufficient number of places in the value of c_x given in [10, 11]). The same problem arose earlier in [13]. So, the leading edges which have been constructed, in spite of the appreciable differences from the generatrices in [10, 11] as regards their shape, reproduce almost all the difference in the c_x values of wedges and optimal bodies. A comparison of Table 1 with Fig. 2(a) shows that the advantages in c_x found for the versions calculated in [10, 11] were far from maximal. Actually, while the maximum advantage with respect to $c_{x\,\mathrm{in}}$ [10, 11] did not exceed 0.66%, according to Fig. 2(a), the maximal $\delta c_x^m > 1.3\%$. Taking into account what has been said above, this corresponds to $\delta c_x \simeq 1.5\%$.

As can be seen from Fig. 2(a), the maximal δc_x^m are obtained in the case of a hypersonic flow past sufficiently thick bodies ($\tau \geqslant 0.2$–0.3). Dissociation and ionization which, in the case of air, are inevitable in such cases necessitate the use of more complex thermodynamics than the thermodynamics of a perfect gas with $\kappa = 1.4$. The perfect gas

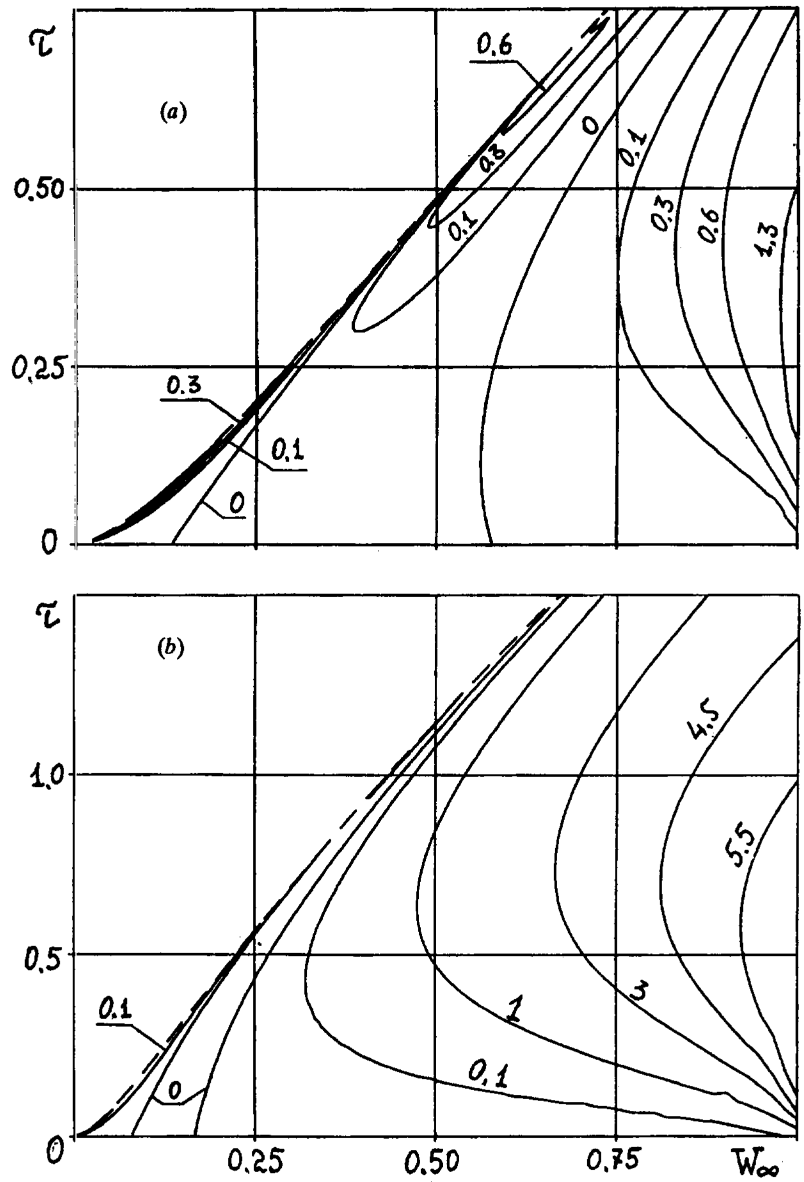

Fig. 2.

model with small κ can, nevertheless, provide an idea of the direction of the effect of these processes. The results of calculations carried out with this aim for $\kappa = 1.1$ are shown in Figs 2–4(b) which are analogous to Figs 2–4(a). A comparison of these figures shows that a reduction in κ leads to a narrowing of the domain D^+ and a reduction in it of the deflection angles and the advantage with respect to c_x. Conversely, in the part D^-, which corresponds to high velocities, both the deflection angles and δc_x^m increase. When $\kappa = 1.1$, the maximum value of δc_x^m reaches 6% here. The possibility of the rapid determination of the dependence of δc_x^m on W_∞ or V_∞, τ, κ, etc. is one of the main results of the approach which has been developed. It has already been noted that the lack of such information prevented Shipilin [10, 11] from carrying out calculations with more advantageous versions. The almost optimal contours in [8] are still less advantageous. In passing, we note that the profiling method proposed in [8], in spite of the self-criticism in [14], is, in the final result, equivalent to the method in [10, 11].

In concluding, we shall discuss the basic differences between the generatrices constructed above and contours which are optimal in the strict sense of this term. In D^-, this difference is not in the least associated with small additional corner points which, moreover, were not taken into account either in [8] or in [10, 11]. The non-optimality of the generatrices constructed is associated in the first place with the fact that, in the scheme in Fig. 1(d), the pencil of rarefaction waves within the limits of *iwf* is not

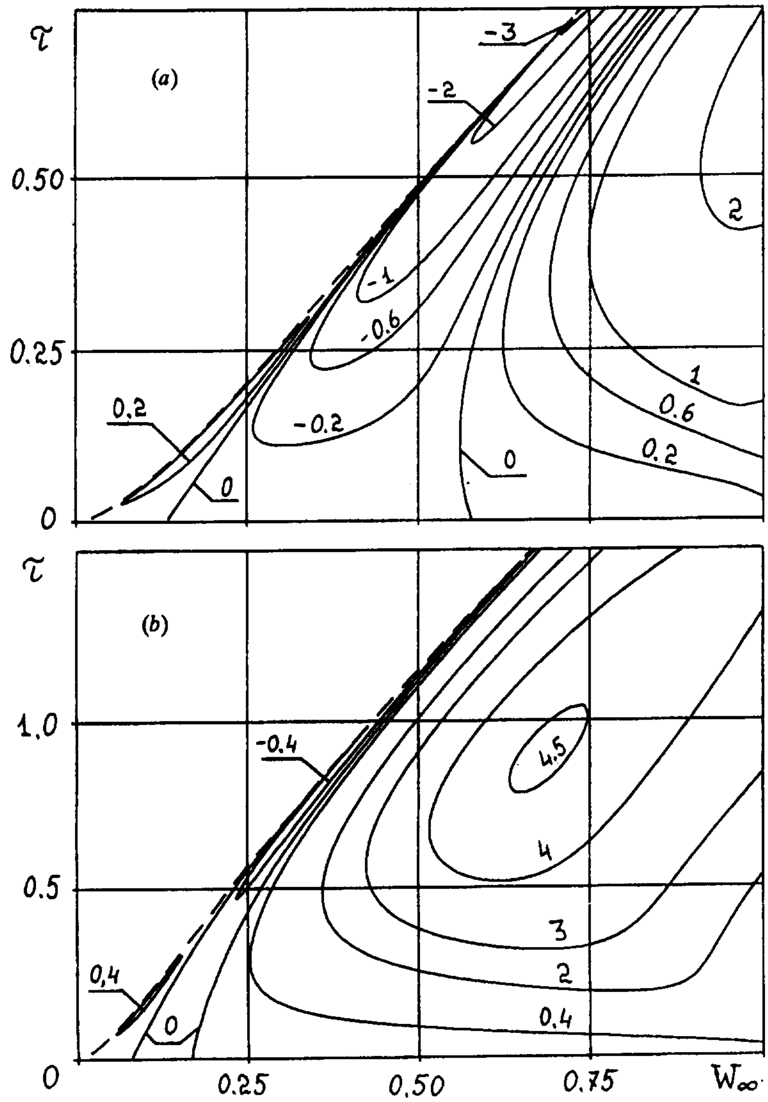

Fig. 3.

reflected from the shock wave. When account is taken of this fact in the expression for $\Delta\chi$ by including terms of higher order than quadratic, this leads to a state of affairs where a small part of the pencil is reflected from the shock wave. In this case, the negative effect of the reflected compression wave ($\lambda < 0$) will be slightly overlapped by the reduction in χ due to the shift to the left of the corner point d. It follows from Table 1 that, in the case of appreciable δc_x^m, the effect of taking account of higher-order terms in $\Delta\chi$ can lead to an additional reduction in c_x by approximately a further $\delta c_x^m/10$.

In D^+, the difference between the optimal contour and that shown in Fig. 1(g) is of a more fundamental nature. In fact, on applying variation in ε-bands to it, we find that the optimal generatrix must have corners at points d_1 and d_2. The corner at d_2, which is associated with reflection from the weak shock wave dw, is unimportant. In contrast to this, the corner point at d_1 will be of the same order as the corner point at d as its occurrence is attributable to reflection from the main shock wave. Allowance for this fact leads to the scheme shown in Fig. 1(h). In this scheme, the rarefaction wave ($\lambda > 0$) reflected from iw is incident as a whole on the contour being optimized and the dimensionless length of the segments hf and d_1d are quantities of the same order of magnitude as the breaks. This, however, does not enable us to assert that the additional reduction in c_x associated with the transition to the new scheme will be of the order of δc_x^m.

Finally, everywhere above, only those deviations of the required generatrices from straight lines were permitted which did not lead to the occurrence of local subsonic zones. In particular, the change in χ

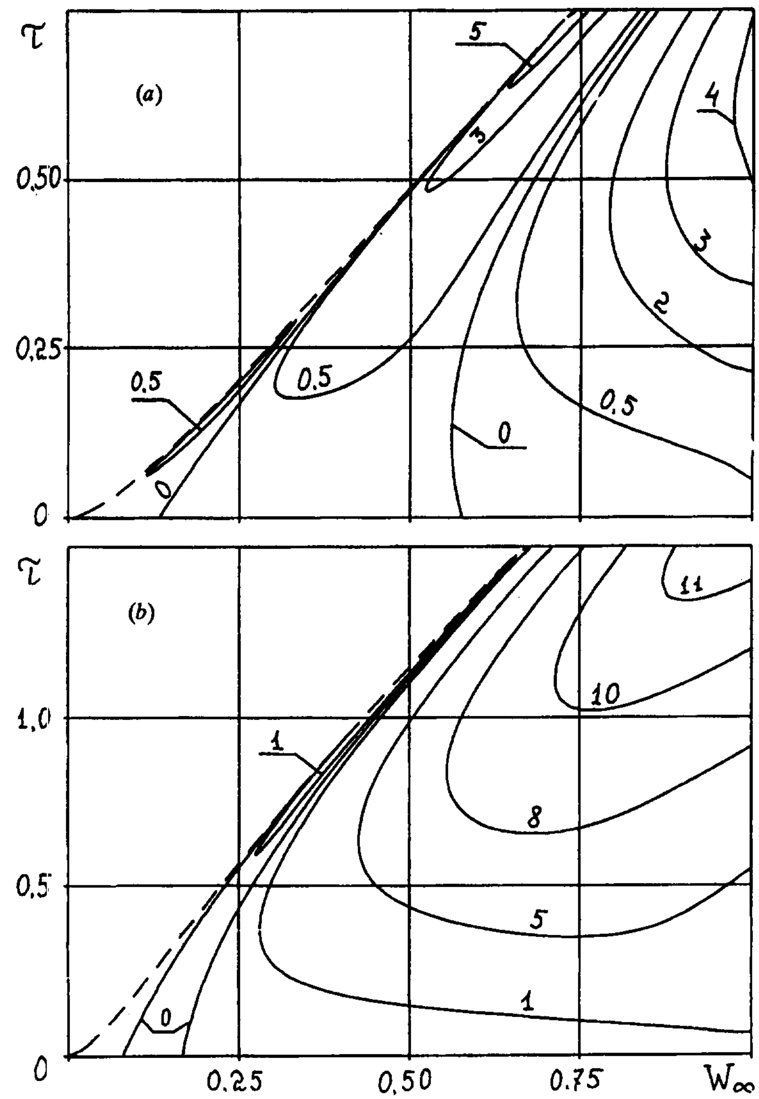

Fig. 4.

Table 1

V_∞	$\tau \times 10^4$	$\Delta\vartheta_d \times 10^4$	$\Delta\vartheta_d''' \times 10^4$	$\delta c_x \times 10^3$	$\delta c_x''' \times 10^3$	$\Delta\delta c_x \times 10^2$
1,4	2128	31	28	20	16	20
1,6	3858	47	30	38	34	9
2,1	4369	209	160	118	107	10
2,3	2069	172	151	299	305	-2
2,3	4802	499	321	656	586	11
2,3	7237	299	208	153	141	8
2,4	7483	526	327	429	394	8

was not investigated when a leading end face was introduced past which flow takes place with a separated shock wave. An attempt to solve this problem within the approximation of Newton's law of resistance suggests that, when almost optimal contours (or optimal contours, like a "sonic" wedge) exist past which there is a flow with an attached shock wave close to the upper boundary of D, that is, close to the dashed curve in Figs 2–4, generatrices with a leading end face are optimal. As V_∞ increases, the width of the part D which is "chopped off" by such solutions, expands. When $M_\infty \gg 1$ and $\tau > 1$, the "sonic" wedge and the wedge which corresponds to the left "null" isoline in Figs 2–4 will certainly not be optimal. Here, the inequality from (2.15) as well as the inequality $dx^w/dy \geqslant 1/\sqrt{3}$, that is the Legendre condition, which, in Newton's approximation must be satisfied on the optimal generatrix [1–3], turn out not to be the sufficient conditions but only the necessary conditions of optimality. According to [15], the introduction of a small end face in the above-mentioned approximation at any and not only at the leading point of the optimal contour gives the inequality: $dx^w/dy \geqslant 1$, which is stronger than the Legendre condition. There is an exactly analogous inequality for the full system of equations of gas dynamics, but the possibility of a flow with an attached shock wave is not determined by $\tau^*(M_\infty)$, which was mentioned at the start of the paper.

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